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Anomalous Higgs interactions in dimensional deconstruction

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Recent LHC experiments have revealed that Higgs is light. As an interesting candidate to accommodate light Higgs, in this paper we adopt the scenario of dimensional deconstruction, where Higgs is regarded as a pseudo-Nambu–Goldstone boson. Though the scenario is formulated in ordinary four-dimensional space-time, it may also be interpreted as "latticized" gauge–Higgs unification. We point out that in this scenario Higgs interaction with the matter field is anomalous, i.e. its coupling deviates from what the standard model predicts. The interplay between the periodicity of physical observables in the Higgs field and the violation of translational invariance along the extra space due to the latticization is argued to play an essential role to get the anomalous interaction. Though the predicted anomalous Higgs interaction has much similarity to the one in the gauge–Higgs unification, in the case of dimensional deconstruction the anomaly exists even if we do not introduce a bulk mass term for the chiral fermion realized by orbifolding, in clear contrast to the case of gauge–Higgs unification. It in turn means that the anomaly goes away in the continuum limit of the extra space.

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1. Introduction

In spite of the great success of the LHC experiments [1,2] in discovering the Higgs particle, we still do not know whether the discovered scalar particle is what we expect in the standard model or a particle some theory of physics beyond the standard model (BSM) has in its low energy effective theory. In other words, we do not have any conclusive argument for the origin of the Higgs.

The experiments, however, have provided us with very important information about the Higgs particle: the observed Higgs mass $M_H = 126\,\mathrm{GeV}$ is of the order of the weak scale M_W. Namely, the Higgs has turned out to be "light," which suggests that the quartic self-coupling of the Higgs λ is of $\mathcal{O}(g^2)$ (g: gauge coupling constant) and, therefore, is handled by the gauge principle.

Among possible BSM models, we can pick up a few candidates where the Higgs self-coupling is handled by gauge interactions. The first candidate is MSSM (minimal supersymmetric standard model), where the coupling λ comes only from the D-term contribution, leading to $M_H \leq \cos 2\beta M_Z$ at the classical level, though a sizable quantum correction is expected to explain the observed Higgs mass assuming larger SUSY-breaking mass scale.

There is another candidate BSM: gauge–Higgs unification (GHU) [3–6]. In this scenario, the Higgs field is identified with the extra space component of the higher-dimensional gauge field. For instance,

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in the simplest five-dimensional (5D) case, the higher-dimensional gauge field is decomposed as

$$A_M = (A_\mu, A_y) \quad (\mu = 0, 1, 2, 3), \tag{1.1}$$

where A_μ just behaves as the ordinary 4D gauge field, while (the Kaluza–Klein (KK) zero mode of) the extra space component A_y is identified as the Higgs field. The fact that the Higgs is originally a gauge boson in GHU provides a new type of solution for the hierarchy problem relying on higher-dimensional gauge symmetry, thus opening a new avenue for BSM theories [7]. In this scenario, the Higgs self-coupling is handled by the gauge principle, simply because Higgs is originally a gauge field. Interestingly, in a 6D GHU on a two-dimensional orbifold T^2/Z_3, an attractive relation $M_H = 2M_W$ holds at the classical level [8] and it has been pointed out that the quantum correction to this relation is calculable as a UV-finite value [9], just as in the case of MSSM.

It is worth noticing that both MSSM and GHU have been proposed in order to solve the hierarchy problem: the problem of quadratically divergent quantum correction to the Higgs mass. There also exist other types of well-discussed BSM theories formulated in 4D space-time, proposed in order to solve the hierarchy problem, which have a close relationship with GHU. Namely, the scenarios of (i) dimensional deconstruction (DD) [10,11], and (ii) little Higgs (LH): see Ref. [12] and references therein. Basically, in both of these scenarios Higgs is regarded as a pseudo-Nambu–Goldstone boson (PNGB) and therefore is non-linearly realized in the form $e^{i\frac{\phi}{f}}$, where ϕ denotes the PNGB treated as the Higgs field and f corresponds to its "decay constant." Let us note that this leads to an important consequence that physical observables are periodic in the Higgs field ϕ in these scenarios. Similar periodicity also exists, as we will discuss below, in the scenario of GHU. As long as the global symmetry is exact the Higgs mass vanishes, suggesting a light Higgs.

As further circumstantial evidence of the mutual relationship between GHU and DD or LH, it may be pointed out that both have shift symmetries, i.e. invariance under the transformations

$$A_y \to A_y + \partial_y \lambda \text{ (for GHU)}, \quad \phi \to \phi + c \text{ (for DD, LH)}, \tag{1.2}$$

where the former is nothing but a higher-dimensional gauge transformation (in the simplest U(1) 5D GHU), and the latter is some global transformation with parameter c, like a phase transformation in the simplest U(1) global symmetry. These symmetries strictly forbid the presence of local operators responsible for the masses of A_y and ϕ, thus solving the hierarchy problem at quantum level.

The mutual close relation becomes manifest and more solid once we realize that the DD scenario may be regarded as a sort of GHU, where the extra space is latticized to several lattice points and $e^{i\frac{\phi}{f}}$ is regarded as the product of "link variables" (or Wilson loop) in a lattice gauge theory, as we will discuss later. It is also worthwhile noting that the scenario of the LH was inspired by the DD scenario.

After the discovery of the Higgs, the main focus of particle physics may now be on checking whether its interactions are what the SM predicts or not. If some deviations from the predictions made by the SM are found by precision tests of the Higgs interactions it will clearly signal the presence of new physics. Precision tests of the Higgs interactions should be one of the main purposes of the proposed International Linear Collider (ILC) experiment.

In this paper we discuss such "anomalous Higgs interactions," i.e. the Higgs couplings which deviate from what the SM predicts.

All the BSM scenarios mentioned above extend the Higgs sector or give new interpretations of the origin of the Higgs, since these were devised in order to solve the hierarchy problem of the Higgs sector relying on some symmetries. Thus it may not be surprising even if these scenarios predict anomalous Higgs interactions.

In the literature there exist works on anomalous Higgs interactions whose contents have some overlap with the present paper. In [13], model-independent analyses of anomalous Higgs interactions have been made by use of a low energy effective Lagrangian with respect to gauge and Higgs fields in a broader class of BSM models, "strongly interacting light Higgs." In addition to the model-independent analyses, the authors have discussed "anomalous" Yukawa coupling in the scenario of the "holographic composite Higgs model," which has a close relationship with the GHU and DD scenarios. Their result, Eq. (42) in their paper, however, is a little different from our result, Eq. (5.3), obtained below for the DD scenario, though both are described by trigonometric functions.

Also, concerning the GHU scenario formulated on the Randall–Sundrum (R–S) type 5D space-time, the anomalous Higgs couplings with $W^{\pm}$ and Z^0 gauge bosons have been discussed in [14–17], and the anomalous Higgs self-couplings have been discussed in [18,19]. Interestingly, the anomalous Higgs couplings with $W^{\pm}$ and Z^0 gauge bosons in the GHU obtained in these papers show the same behavior described by the cosine function as we obtain for the DD scenario, Eq. (5.3). One possible reason for this coincidence of the results in two different models is that in both cases the translational invariance along the extra space is broken by the property of the extra space itself, i.e. by the warp factor in the case of GHU on the R–S background and by the latticization in our present paper. (The importance of the violation of translational invariance in the context of anomalous Higgs interactions is discussed below.)

In our previous paper [20] we have discussed anomalous Higgs interactions in the scenario of GHU formulated on a flat 5D space-time compactified on an orbifold S^1/Z_2. The behavior of the obtained anomalous Yukawa coupling, Eq. (1.5) below, is a little different from those in the GHU on the R–S background or that obtained in the present paper.

Probably one of the most important observations in the analysis concerning the anomalous interactions in the previous paper [20] is that the anomaly is an inevitable consequence of the following two properties of the theory: (a) the periodicity of the physical observables in the Higgs field, and (b) the violation of the translational invariance along the extra space. Let us discuss a little why these properties are essential to get the anomaly and how they appear in the GHU scenario.

First, concerning issue (a), we should note that in GHU, the Higgs field may be interpreted as a Wilson loop phase or a sort of AB (Aharonov–Bohm) phase, at least in 5D space-time where the extra space is a circle, a non-simply connected space. Namely, the circle allows the penetration of magnetic flux inside itself and the zero mode of A_y is regarded as a vector potential generated by the magnetic flux. Thus the vacuum expectation value (VEV) of Higgs, though it is just a constant, is not a pure gauge and cannot be gauged away. On the other hand, as long as the Higgs field appears through a phase, we naturally expect that physical observables in this theory are all periodic in the Higgs field.

Typical periodic functions are trigonometric functions. In fact, for the KK zero modes of lighter quarks (say, the quarks of first and second generations), their masses behave as sine functions of the VEV of the Higgs field v [14–20]:

$$m(v) \propto \sin\left(\frac{g_4}{2}\pi R v\right), \tag{1.3}$$

where g_4 is the 4D gauge coupling and R is the radius of the circle, the extra space. Since Higgs interaction is expected to be obtained by the replacement $v \to v + h$ with h being the physical Higgs field, the Yukawa coupling (to be precise, the "diagonal part" of the Yukawa coupling matrix in the base of KK modes for each flavor of quarks) is given by the first derivative of the "mass function"

$m(v)$:

$$f = \frac{dm(v)}{dv} \propto \cos\left(\frac{g_4}{2}\pi R v\right), \tag{1.4}$$

which is no longer constant as in the SM, since the mass function is not linear in v. Thus the Yukawa coupling for light quarks predicted by GHU, f_{GHU}, deviates from that in the SM, f_{SM}, as [20]

$$\frac{f_{\mathrm{GHU}}}{f_{\mathrm{SM}}} \simeq x \cot x \quad \left(x \equiv \frac{g_4}{2}\pi R v\right), \tag{1.5}$$

which even vanishes for a specific value of the VEV, $x = \frac{v}{2}$ [14–17], though the ratio reasonably approaches unity for small x, since in this limit $M_W \ll \frac{1}{R}$ and the SM is expected to be recovered as the low energy effective theory of GHU.

One remark is in order here. In a realistic GHU model, to make the theory chiral, we adopt an orbifold as the extra space. In the case of 5D GHU, the orbifold is S^1/Z_2. In this framework, a "Z_2-odd bulk mass term"

$$\epsilon(y) M \bar{\psi}\psi \tag{1.6}$$

is allowed, being Z_2 invariant, where $\epsilon(y)$ is the sign function of the extra space coordinate y: $\epsilon(y) = \pm 1$ depending on the sign of y. This mass term causes the localization of mode functions of chiral fermions at different fixed points of the orbifold depending on their chiralities, and the overlap of their mode functions is exponentially suppressed, thus naturally leading to hierarchically small (zero mode) quark masses for lighter generations. The sine function Eq. (1.3) is known to be obtained as the result of the presence of this bulk mass term as we discuss below. Let us note that this Z_2-odd bulk mass term, behaving as a sort of kink solution of the scalar field, clearly violates the translational invariance along the extra space.

The argument above seems to suggest that just the periodicity leads to the anomalous Yukawa coupling. Then what role does the property or condition (b) play to get the anomaly? Actually, the periodicity (a) does not necessarily lead to such a non-linear mass function as in Eq. (1.3). In fact, in [20] we have argued that for the KK zero mode of a heavy quark like the top quark, the mass function is just a linear function of v, just as in the SM, while the periodicity is still guaranteed by the replacement of the KK zero mode by the first KK mode at the "crossing point" of the mass spectrum at $x = \frac{\pi}{2}$. The essence of the argument is that for a heavy quark the Z_2-odd bulk mass M can be switched off and the translational invariance along the extra space is not violated (to be more precise, the absolute values of the extra space momenta or KK modes are preserved even under the presence of fixed points), and the mass function for the zero mode is a "normal" linear function of v, just as in the original argument in the case of S^1 compactification [7]. At the crossing point the KK zero mode and the first KK mode do not mix with each other due to the translational invariance, i.e. the conservation of extra space momentum p_y.

What happens if we switch on a small bulk mass M? Now the mixing between zero and first KK modes arises, the degeneracy of the mass spectrum at the level crossing is lifted a little, and the linear function is slightly modified, accordingly. When the bulk mass becomes sizable, as in the case of lighter quarks, the mode function of the zero mode is considerably modified and we finally obtain the sine function as shown in Eq. (1.3). We thus realize that anomalous interaction is the inevitable consequence of the interplay of the periodicity and the violation of the translational invariance, as mentioned in (a) and (b) above.

The purpose of this paper is to discuss anomalous Higgs interaction in the scenario of dimensional deconstruction (DD) [10,11].

As has been already mentioned above, in the original proposal of the DD scenario [10,11] the Higgs is a PNGB composed of a fermion and anti-fermion paired by strong interactions, just as the pions are composed of pairs of quarks and anti-quarks in QCD. The specific feature of the model is that after the confinement by the strong interactions the remaining (weak) gauge symmetries are of the form of the direct product of the same type of gauge group: $\mathrm{SU}_1(\mathrm{m}) \times \mathrm{SU}_2(\mathrm{m}) \times \cdots \times \mathrm{SU}_N(\mathrm{m})$, as is schematically displayed by the "moose diagram."

From a different point of view, the DD scenario can be interpreted as a sort of GHU scenario, where the extra space is "latticized," N being the number of lattice sites. In fact, the non-linear realization of the Higgs field ϕ,

$$U = e^{i\sqrt{N}\frac{\phi}{f}}, \tag{1.7}$$

just corresponds to the Wilson loop in GHU.

Such a close relation of DD with GHU strongly suggests that similar anomalous Higgs interactions to those in GHU are expected in the DD scenario as well. We show in this paper that this is really the case.

We can easily understand that conditions (a) and (b) to get anomalous interaction mentioned above are also met in the DD scenario we are now interested in. First, condition (a), i.e. the periodicity in the Higgs field, also exists in this theory, since the Higgs field is non-linearly realized as a sort of phase factor, as is seen in Eq. (1.7). Secondly, it is clear that the translational invariance is violated by the fact that the extra space is latticized, once DD is understood as a latticized 5D GHU. Thus it is almost promising that we get anomalous Higgs interactions in the scenario of DD.

However, we should also note that there is a qualitatively distinct difference in the anomalous interaction present in the DD scenario from the one in GHU. Namely, it is because of the violation of the translational invariance by latticizing the extra space, not because of the Z_2-odd bulk mass for fermion as in the case of GHU. As far as the violation is due to the property of the space-time itself on which the theory is constructed, the anomalous interactions should arise not only in the sector of matter fermion but basically in every sector of the theory. The situation, in such a sense, may be similar to the case of GHU formulated on the R–S background, where the translational invariance is violated by the presence of the warp factor $e^{-\kappa|y|}$ and anomalous interactions appear not only in the fermionic sector but also in the gauge boson sector as well [14]. This, on the other hand, suggests that the anomaly goes away in the limit $a \to 0$ (a: lattice spacing), unless there is no other source of violation of translational invariance.

In this paper, we take the attitude that the scenario of DD is equivalent to a latticized GHU and study the anomalous Higgs interaction in the GHU with latticized extra space. We will discuss the anomalous Higgs interactions first in 5D scalar QED, as a toy model to see the essence of the mechanism, and next in 5D QED with matter fermion, both with latticized extra space as its compactified extra space [21,22].

After all, Higgs interactions are divided into two categories, i.e. anomalous versus normal, and to see which category is chosen by Nature is quite important in order to conclude whether physics BSM is realized or not and, if it is ever realized, which type of BSM is selected.

2. Five-dimensional gauge theory with latticized extra space

Before discussing five-dimensional (5D) scalar QED and 5D QED, we consider here a generic 5D gauge theory where the extra dimension is compactified S^1 of radius R and circumference $L = 2\pi R$, which is latticized to N lattice sites with extra space coordinates $y_i\,(i = 1, \ldots, N)$.

For a generic matter field $\psi(x^\mu, y_i)$, a local gauge transformation is given as

$$\psi(x^\mu, y_i) \to \psi'(x^\mu, y_j) = g(x^\mu, y_j)\psi(x^\mu, y_j), \tag{2.1}$$

where $g(x^\mu, y_j)$ is a member of the gauge group $\mathcal{G}$. In the limit of $N \to \infty$ this transformation reduces to a 5D local gauge transformation. On the other hand, we may regard it as the 4D local gauge transformation whose gauge group is a direct product,

$$G_1 \times G_2 \times \cdots \times G_N, \tag{2.2}$$

where each G_j, with the group element $g(x^\mu, y_j)$, belongs to the same group $\mathcal{G}$. Equation (2.2) is equivalent to the gauge symmetry shown by the "moose diagram" in the original DD scenario [10,11], where G_i are "weak" gauge symmetries remaining after the confinement due to the strong forces. In this way, we can confirm that the scenario of DD is equivalent to the GHU where the extra space is latticized.

Hereafter, we change the notation of the field as

$$\psi(x^\mu, y_j) \to \psi_i(x^\mu). \tag{2.3}$$

The fields $\psi_i(x^\mu)(i = 1, 2, \ldots, N)$ may also be regarded as N pieces of 4D fields. Because of the S^1 compactification, there is a periodic boundary condition as follows:

$$\psi_{N+i}(x^\mu) = \psi_i(x^\mu). \tag{2.4}$$

The "derivative" along the extra space is given by a difference,

$$\partial_y \psi_i(x^\mu) \equiv \frac{\psi_{i+1}(x^\mu) - \psi_i(x^\mu)}{a}, \tag{2.5}$$

where a (> 0) is the distance between neighboring sites (lattice spacing) satisfying

$$L \equiv 2\pi R = Na. \tag{2.6}$$

The covariant derivative along 4D space-time is just an ordinary one:

$$D_\mu \psi_i = \partial_\mu \psi_i - igA_{i\mu}\psi_i, \tag{2.7}$$

where $A_{i\mu}$ is the gauge field of G_i and g is the gauge coupling constant.

The covariant derivative along the extra dimension is given by

$$D_y \psi_i = \frac{\psi_{i+1} - U_i\psi_i}{a}, \tag{2.8}$$

where

$$U_i(x^\mu) = e^{igaA_{iy}(x^\mu)} \tag{2.9}$$

is a "link variable" (Wilson line) to be introduced for gauge covariance, which connects the ith and $(i+1)$th sites. $A_{iy}(x^\mu)$ corresponds to the extra space component of gauge field $A_y(x^\mu, y)$ in GHU. In order to guarantee the gauge covariance, the link variable should transform under the local gauge transformation as follows:

$$U_i \to U_i' = g_{i+1}U_i g_i^\dagger, \tag{2.10}$$

where g_i, g_{i+1} are group elements of G_i, G_{i+1}, respectively. Namely, U_i behaves as a bifundamental representation of (G_i, G_{i+1}).

We also need the covariant derivative for a link variable U_i in order to get the kinetic term for A_{iy}. Since U_i behaves as the bifundamental representation of (G_i, G_{i+1}), its 4D covariant derivative is given as

$$D_\mu U_i = \partial_\mu U_i - igA_{i+1,\mu}U_i + igU_iA_{i\mu}. \tag{2.11}$$

3. Five-dimensional scalar QED

For illustrative purposes, to see the mechanism of anomalous Higgs interaction, we first take a toy model, i.e. 5D scalar QED on the latticized extra space [21], following the prescription discussed in the previous section.

The model is composed of a 5D scalar electron $\phi_i(x^\mu)$ with electric charge $-e$ and the 5D photon $(A_{i\mu}(x^\mu), A_{iy}(x^\mu))$. The lightest 4D field, corresponding to the KK zero mode in GHU, of $A_{iy}(x^\mu)$ is identified with the Higgs field and is supposed to have a VEV. Thus the scalar electron has masses due to the VEV, though the gauge symmetry is not broken in this U(1) Abelian gauge theory. We expect in a realistic model with non-Abelian gauge symmetry to incorporate the standard model, gauge symmetry is broken through the Hosotani mechanism [4–6].

The 4D Lagrangian, which corresponds to the integral over y of the 5D Lagrangian in GHU, is given by

$$\mathcal{L} = a\sum_{i=1}^{N}\left\{-\frac{1}{4}F_i^{\mu\nu}F_{i\mu\nu} + \frac{1}{(ae)^2}(D^\mu U_i)^* D_\mu U_i + (D^\mu\phi_i)^* D_\mu\phi_i - (D_y\phi_i)^* D_y\phi_i - m^2\phi_i^*\phi_i\right\}, \tag{3.1}$$

where

$$F_i^{\mu\nu} = \partial^\mu A_i^\nu - \partial^\nu A_i^\mu, \tag{3.2}$$

$$U_i = e^{-iaeA_{iy}}, \tag{3.3}$$

and covariant derivatives are given as

$$\begin{aligned} D^\mu U_i &= \partial^\mu U_i + ieA_{i+1}^\mu U_i - ieU_i A_i^\mu \\ &= -i(ea)\left(\partial^\mu A_{iy} - \frac{A_{i+1}^\mu - A_i^\mu}{a}\right)U_i = -i(ea)\left(\partial^\mu A_{iy} - \partial_y A_i^\mu\right)U_i, \end{aligned} \tag{3.4}$$

$$D^\mu\phi_i = \partial^\mu\phi_i + ieA_i^\mu\phi_i, \tag{3.5}$$

$$D_y\phi_i = \frac{\phi_{i+1} - U_i\phi_i}{a}. \tag{3.6}$$

So far the charge e and all fields are regarded to be 5D coupling and fields, respectively. We later introduce a 4D electric charge e_4.

3.1. Kaluza–Klein mode expansion

We now perform a "discretized Fourier transform" for each 5D field in order to get 4D mass eigenstates.

First, let us note that although the translational invariance along the extra space is violated by latticization, there still remains a symmetry in the theory under the following discrete transformation:

$$D: y_i \to y_{i+1}, \quad \text{i.e.} \quad \phi_i \to \phi_{i+1}, \quad \text{etc.} \tag{3.7}$$

On the other hand, repeating D N times should be the identity transformation. Thus, the eigenvalues of D should be

$$(\omega_N)^n \quad \left(\omega_N \equiv e^{i\frac{2\pi}{N}}, \quad n = 0, 1, 2, \ldots, N-1\right). \tag{3.8}$$

Thus, the KK mode functions (vectors with N elements) can easily be found without solving eingenvalue equation for 4D mass eigenvalues:

$$\begin{pmatrix} 1 \\ (\omega_N)^n \\ (\omega_N)^{2n} \\ \vdots \\ (\omega_N)^{(N-1)n} \end{pmatrix}. \tag{3.9}$$

Using these eigenvectors we easily get a (discretized) Fourier series expansion of each field:

$$A_{i\mu}(x^\mu) = \frac{1}{\sqrt{L}} \sum_{n=0}^{N-1} A_\mu^{(n)}(x^\mu)(\omega_N)^{in}, \tag{3.10}$$

$$A_{iy}(x^\mu) = \frac{1}{\sqrt{L}} \sum_{n=0}^{N-1} A_y^{(n)}(x^\mu)(\omega_N)^{in}, \tag{3.11}$$

$$\phi_i(x^\mu) = \frac{1}{\sqrt{L}} \sum_{n=0}^{N-1} \phi^{(n)}(x^\mu)(\omega_N)^{in}, \tag{3.12}$$

where $\phi^{(n)}(x^\mu)$ etc. are 4D fields with proper canonical mass dimension ($d = 1$) of KK mode n. The KK zero mode of A_{iy}, $A_y^{(0)}$, is identified with the Higgs field. The reality of the gauge fields $A_{i\mu}$, A_{iy} is guaranteed by

$$A_\mu^{(N-n)} = \left(A_\mu^{(n)}\right)^*, \quad A_y^{(N-n)} = \left(A_y^{(n)}\right)^*. \tag{3.13}$$

Let us note that the KK zero mode of $A_{i\mu}$, i.e. $A_\mu^{(0)}$, appears at each lattice site as $\frac{1}{\sqrt{L}}A_\mu^{(0)}(x^\mu)$—see Eq. (3.10). Thus, the 4D electric charge e_4, which is nothing but the coupling constant of $A_\mu^{(0)}(x^\mu)$ with the scalar electron, is given by

$$e_4 = \frac{e}{\sqrt{L}} = \frac{e}{\sqrt{2\pi R}}, \tag{3.14}$$

just as in the case of GHU.

3.2. *Four-dimensional mass eigenvalues*

Getting mass eigenstates, we now calculate the 4D mass eigenvalues of KK modes.

3.2.1. *Four-dimensional masses in the gauge–Higgs sector*

First we discuss the gauge–Higgs sector, i.e. the sector of $A_\mu^{(n)}$ and $A_y^{(n)}$. Note that this sector does not acquire the masses due to the VEV v of the Higgs field $A_y^{(0)}$. We should also note that except for the Higgs field $A_y^{(0)}$, all non-zero KK modes of A_y are absorbed by a sort of Higgs mechanism to the corresponding massive KK modes of A_μ.

Substituting the mode expansion (3.10) and (3.11) in the relevant part of the Lagrangian (3.1),

$$a \sum_i \frac{1}{(ae)^2} (D^\mu U_i)^* D_\mu U_i, \tag{3.15}$$

and performing the sum over i we get

$$\sum_{n=0}^{N-1} \left(\partial^\mu A_y^{(n)*} - \frac{(\omega_N)^{-n} - 1}{a} A_\mu^{(n)*}\right) \left(\partial_\mu A_y^{(n)} - \frac{(\omega_N)^n - 1}{a} A_\mu^{(n)}\right). \tag{3.16}$$

Here we have used the orthonormal condition

$$\sum_{i=0}^{N-1} (\omega_N)^{in} (\omega_N)^{im} = N\delta_{n+m}, \tag{3.17}$$

where $n+m$ is in mod N. By the rephasing of the fields $A_\mu^{(n)} \to -i(\omega_N)^{-\frac{n}{2}} A_\mu^{(n)}$, the coefficient $\frac{(\omega_N)^n - 1}{a}$ is made real:

$$\frac{(\omega_N)^n - 1}{a} \to -i\frac{(\omega_N)^{\frac{n}{2}} - (\omega_N)^{-\frac{n}{2}}}{a} = \frac{2\sin\left(\frac{n}{N}\pi\right)}{a}. \tag{3.18}$$

In this way it is clear that for the sector of non-zero KK modes a Higgs-like mechanism is operative and the 4D mass eigenvalues of massive gauge bosons are given as

$$m_n = \frac{2\sin\left(\frac{n}{N}\pi\right)}{a}. \tag{3.19}$$

Note that in the "continuum limit" $N \to \infty (a \to 0)$ keeping L intact, the mass eigenvalue reduces (for low-lying KK modes, $\frac{n\pi}{N} \ll 1$) to

$$m_n \to 2\frac{n}{Na}\pi = \frac{2n\pi}{L} = \frac{n}{R} \quad (L = Na = 2\pi R), \tag{3.20}$$

which is nothing but the well-known KK masses in higher-dimensional theories. These eigenvalues and the eigenvectors given in Eq. (3.9) are just the same as those in a system of coupled harmonic oscillators: the system of springs and balls.

3.2.2. *Four-dimensional masses of matter field*

Next, we discuss the mass eigenvalues of the scalar electron ϕ. Again by substituting the mode expansion (3.12) in the part relevant for the mass-squared term and replacing A_i by its VEV v, we get

$$-a\sum_i \left\{(D_y\phi_i)^* D_y\phi_i + m^2\phi_i^*\phi_i\right\} = -\sum_{n=0}^{N-1} m_n^2 \left|\phi^{(n)}\right|^2, \tag{3.21}$$

where

$$m_n^2 = \frac{1}{a^2}\left|(\omega_N)^n - e^{-iae_4 v}\right|^2 + m^2 = \left\{\frac{2}{a}\sin\left(\frac{n\pi}{N} + \frac{ae_4 v}{2}\right)\right\}^2 + m^2. \tag{3.22}$$

Again, at the continuum limit $|\frac{n\pi}{N} + \frac{ae_4 v}{2}| \ll 1$, which is valid for low-lying KK modes, the m_n reduces to

$$m_n^2 \to \left(\frac{n}{R} + e_4 v\right)^2 + m^2, \tag{3.23}$$

recovering the result in the GHU of 5D QED with S^1 compactification [7].

3.3. *The coupling constants of Higgs interaction*

Our main purpose is to investigate whether the Higgs couplings with matter fields show some anomalous behavior. In this section, we thus focus on the Higgs couplings with the scalar electron, which is regarded as the counterpart of the Yukawa coupling in a realistic model, which can incorporate SM.

We have obtained the mass-squared term for the nth KK mode of the scalar electron ϕ—see Eqs. (3.21), (3.22):

$$\left(\left\{\frac{2}{a}\sin\left(\frac{n\pi}{N}+\frac{ae_4 v}{2}\right)\right\}^2+m^2\right)\phi^{(n)}(x^\mu)^*\phi^{(n)}(x^\mu). \tag{3.24}$$

Since the physical Higgs field is nothing but the deviation of the Higgs field from its VEV, the Higgs interactions with ϕ are expected to be obtained by the following replacement in Eq. (3.24):

$$v \to v + h(x^\mu), \tag{3.25}$$

where h denotes the physical Higgs field. As has already been discussed in Sect. 1, in the case of 5D GHU the non-linearity of the mass eigenvalues came from the violation of translational symmetry along the extra space due to the presence of the Z_2-odd bulk mass term for the fermion, and the Yukawa coupling was found to have "off-diagonal" couplings between different KK modes [20]. Thus, the Higgs interactions obtained by the prescription in Eq. (3.25) were argued to represent the "diagonal" couplings between the same KK mode. It is interesting to note that in our model based on the scenario of DD, the non-linearity comes from the fact that in the covariant derivative the Higgs is non-linearly realized from the beginning. Thus Higgs interactions are expected to be diagonal in the base of KK modes, in contrast to the case of GHU.

For instance, the coupling constant of three-point coupling between the Higgs field and the nth KK mode of matter scalar $h\phi^{(n)*}\phi^{(n)}$ is given by the first derivative of the mass-squared m_n^2 in Eq. (3.24), given by Eq. (3.22), with respect to the VEV v,

$$\frac{dm_n^2}{dv}=\frac{2e_4}{a}\sin\left(\frac{2n\pi}{N}+ae_4 v\right). \tag{3.26}$$

Similarly, the coupling constant of four-point coupling $h^2\phi^{(n)*}\phi^{(n)}$ is calculated to be

$$\frac{1}{2}\frac{d^2m_n^2}{dv^2}=e_4^2\cos\left(\frac{2n\pi}{N}+ae_4 v\right). \tag{3.27}$$

We are particularly interested in the coupling constants of the Higgs field with the lightest KK-mode $n_{\rm min}$ of the matter scalar, which has the smallest $|\sin\left(\frac{n\pi}{N}+\frac{ae_4v}{2}\right)|$. The coupling constants of the lightest scalar are given as:

$$\text{3-point coupling}: \quad \frac{2e_4}{a}\sin\left(\frac{2n_{\rm min}\pi}{N}+ae_4 v\right), \tag{3.28}$$

$$\text{4-point coupling}: \quad e_4^2\cos\left(\frac{2n_{\rm min}\pi}{N}+ae_4 v\right). \tag{3.29}$$

We realize that, for instance, the four-point coupling (3.29), which in some sense mimics the Yukawa coupling, being dimensionless, shows an anomalous behavior like the Yukawa coupling in 5D GHU mentioned in Sect. 1 [14–20].

4. Five-dimensional QED

We now work in a little realistic theory, i.e. deconstructed 5D QED [22] with matter fermion (electron) $\psi_i(x^\mu)$ of charge $-e$ and mass m, instead of the scalar electron $\phi_i(x^\mu)$.

Basically, we can just follow the procedure to get the Higgs interaction in the previous toy model. One non-trivial issue in this model, however, is the problem of "flavor doubling," as discussed in [22]. The original cause of the problem is the latticization of the extra space, namely the presence of a second Brillouin zone, as is well known in the case of 4D lattice gauge theories. Such a doubling problem does not exist for the bosonic sector, therefore not in the scalar QED in the previous section.

A possible way out of this problem is to put a "Wilson term," say a bosonic kinetic (covariant derivative) term, in its continuum limit, $D_y D^y$, which behaves as a momentum-dependent mass term and has the effect of lifting the 4D mass of the redundant zero mode in the second Brillouin zone. We will explain in some detail how the doubling problem is evaded.

Before the inclusion of the Wilson term, the relevant part of the 4D fermion mass spectrum of the original Lagrangian reads as

$$
\begin{aligned}
& -a\sum_i \left\{ m\bar{\psi}_i\psi_i - \eta\frac{1}{2a}\left[\bar{\psi}_{i+1}\gamma_5(\psi_{i+1}-U_i\psi_i) - \left(\bar{\psi}_{i+1}-\bar{\psi}_i U_i^\dagger\right)\gamma_5\psi_{i+1}\right]\right\} \\
&\quad = -a\sum_i \left\{ m\bar{\psi}_i\psi_i + \eta\frac{1}{2a}\left[\bar{\psi}_{i+1}\gamma_5 U_i\psi_i + \text{h.c.}\right]\right\},
\end{aligned}
\tag{4.1}
$$

where in the nearest-neighbor hopping term, we have taken the average of "forward and backward" covariant derivatives. A dimensionless parameter η introduced in the hopping term of the equation above will be fixed later.

We now add the Wilson term to (4.1), which, being the form of the second covariant derivative, $D_y D^y$, can be written in latticized form as

$$
\begin{aligned}
& -a\sum_i \eta'\frac{1}{2a}\left(\bar{\psi}_i U_i^\dagger\psi_{i+1} + \bar{\psi}_i U_{i-1}\psi_{i-1} - 2\bar{\psi}_i\psi_i\right) \\
&\quad = -a\sum_i \eta'\frac{1}{2a}\left(\bar{\psi}_i U_i^\dagger\psi_{i+1} + \bar{\psi}_{i+1} U_i\psi_i - 2\bar{\psi}_i\psi_i\right),
\end{aligned}
\tag{4.2}
$$

where we have introduced another dimensionless parameter η', which will also be fixed below. Adding this term to (4.1), we obtain

$$
-a\sum_i \left\{ \tilde{m}\bar{\psi}_i\psi_i + \frac{1}{2a}\left[(\eta+\eta')\bar{\psi}_{i+1,L}U_i\psi_{i,R} - (\eta-\eta')\bar{\psi}_{i+1,R}U_i\psi_{i,L} + \text{h.c.}\right]\right\},
\tag{4.3}
$$

where

$$
\tilde{m} \equiv m - \frac{\eta'}{a}.
\tag{4.4}
$$

It can be argued [22] that the origin of the flavor doubling is the presence of both hoppings $\psi_{i,R} \to \psi_{i+1,L}$ and $\psi_{i,L} \to \psi_{i+1,R}$ with equal weight for $\eta' = 0$. We therefore eliminate one of the hoppings $\psi_{i,L} \to \psi_{i+1,R}$ by setting

$$
\eta' = \eta.
\tag{4.5}
$$

The term (4.3) is now rewritten as

$$
-a\sum_i \left\{ \tilde{m}\bar{\psi}_i\psi_i + \frac{\eta}{a}\left[\bar{\psi}_{i+1,L}U_i\psi_{i,R} + \text{h.c.}\right]\right\} \quad \left(\tilde{m} \equiv m - \frac{\eta}{a}\right).
\tag{4.6}
$$

The remaining η will be fixed shortly.

4.1. Four-dimensional masses of electron

The (discretized) Fourier series expansion of the electron is similar to (3.12) for the scalar electron,

$$\psi_i(x^\mu) = \frac{1}{\sqrt{L}} \sum_{n=0}^{N-1} \psi^{(n)}(x^\mu)(\omega_N)^{in}. \tag{4.7}$$

Then, substituting this expansion in (4.6) and replacing A_i by its VEV v, we get

$$-\sum_{n=0}^{N-1} \left\{ \tilde{m}\bar{\psi}_L^{(n)}\psi_R^{(n)} + \eta(\omega_N)^n \frac{\langle U \rangle}{a} \bar{\psi}_L^{(n)}\psi_R^{(n)} + \text{h.c.} \right\}, \tag{4.8}$$

where $\langle U \rangle = e^{-iae_4 v}$. The term (4.8) can be rewritten as

$$-\sum_{n=0}^{N-1} \left\{ \bar{\psi}^{(n)} \left(\tilde{m} + \frac{\eta}{a} \cos\left(\frac{2n}{N}\pi + ae_4 v \right) \right) \right) \psi^{(n)} - i\frac{\eta}{a} \sin\left(\frac{2n}{N}\pi + ae_4 v \right) \right) \bar{\psi}^{(n)}\gamma_5\psi^{(n)} \right\}. \tag{4.9}$$

In order to eliminate γ_5, we perform a chiral rotation $\psi^{(n)} \to \hat{\psi}^{(n)}$:

$$\hat{\psi}^{(n)} = e^{-i\frac{\theta}{2}\gamma_5}\psi^{(n)} \quad \left(\tan\theta = \frac{\frac{\eta}{a}\sin\left(\frac{2n}{N}\pi + ae_4 v\right)}{\tilde{m} + \frac{\eta}{a}\cos\left(\frac{2n}{N}\pi + ae_4 v\right)} \right), \tag{4.10}$$

to get

$$-\sum_{n=0}^{N-1} m_n \bar{\hat{\psi}}^{(n)}\hat{\psi}^{(n)}, \tag{4.11}$$

with the 4D mass eigenvalues of the fermion m_n given by

$$m_n^2 = 4\left(\frac{\eta}{a} - m\right)\frac{\eta}{a}\sin^2\left(\frac{n}{N}\pi + \frac{ae_4 v}{2}\right) + m^2. \tag{4.12}$$

In order to recover the expected mass spectrum in the continuum limit, $m_n^2 = \left(\frac{n}{R} + e_4 v\right)^2 + m^2$, we fix the remaining parameter η by imposing the following condition:

$$\left(\frac{\eta}{a} - m\right)\eta = \frac{1}{a}. \tag{4.13}$$

Thus we finally obtain

$$m_n^2 = \left\{ \frac{2}{a}\sin\left(\frac{n}{N}\pi + \frac{ae_4 v}{2}\right) \right\}^2 + m^2, \tag{4.14}$$

which coincides with the mass spectrum of the scalar electron, (3.22), and therefore does not suffer from the doubling problem.

4.2. Yukawa coupling

The Yukawa coupling f_{DD} of the fermion with the Higgs field in our DD scenario is calculated by replacing v by $v + h$ in (4.9) and focusing on the term linear in the Higgs field h:

$$-\eta e_4 \sum_i \bar{\psi}^{(n)} \left[\sin\left(\frac{2n}{N}\pi + ae_4 v\right) + i\cos\left(\frac{2n}{N}\pi + ae_4 v\right)\gamma_5 \right] \psi^{(n)} h. \tag{4.15}$$

Moving to the mass eigenstates $\hat{\psi}^{(n)}$ by use of (4.10), the Yukawa coupling now reads:

$$-\eta e_4 \sum_n \frac{1}{m_n} \bar{\hat{\psi}}^{(n)} \left[\tilde{m}\sin\left(\frac{2n}{N}\pi + ae_4 v\right) + i\left(\tilde{m}\cos\left(\frac{2n}{N}\pi + ae_4 v\right) + \frac{\eta}{a}\right)\gamma_5 \right] \hat{\psi}^{(n)} h. \tag{4.16}$$

By imposing (4.13), this reduces to

$$e_4 \sum_n \frac{1}{m_n a} \bar{\hat{\psi}}^{(n)} \left[\sin\left(\frac{2n}{N}\pi + ae_4 v\right) - i\left(\eta^2 - \cos\left(\frac{2n}{N}\pi + ae_4 v\right)\right)\gamma_5\right] \hat{\psi}^{(n)} h. \tag{4.17}$$

It should be noted that in addition to ordinary scalar-type Yukawa coupling with the coupling constant $f_{\rm DD}^{s}$, there appears a pseudo-scalar-type Yukawa coupling with the coupling constant $f_{\rm DD}^{ps}$:

$$f_{\rm DD}^{s} = e_4 \frac{1}{m_n a} \sin\left(\frac{2n}{N}\pi + ae_4 v\right), \tag{4.18}$$

$$f_{\rm DD}^{ps} = -e_4 \frac{1}{m_n a} \left[\eta^2 - \cos\left(\frac{2n}{N}\pi + ae_4 v\right)\right]. \tag{4.19}$$

We may naively expect that the first derivative of the mass eigenvalue provides the Yukawa coupling as in the standard model. However, it is known to reproduce only the scalar-type Yukawa coupling:

$$f_{\rm DD}^{s} = \frac{dm_n}{dv} = e_4 \frac{\left(\frac{1}{a}\right)\sin\left(\frac{2n}{N}\pi + ae_4 v\right)}{\sqrt{\left\{\frac{2}{a}\sin\left(\frac{n}{N}\pi + \frac{ae_4 v}{2}\right)\right\}^2 + m^2}}. \tag{4.20}$$

In a realistic model incorporating the standard model, the masses of the KK zero mode should be provided only through the spontaneous breakdown of gauge symmetry. Thus, ignoring the bulk mass m, the condition (4.13) implies that

$$\eta^2 = 1, \tag{4.21}$$

and the mass eigenvalues just reduce to

$$m_n = \frac{2}{a}\sin\left(\frac{n}{N}\pi + \frac{ae_4 v}{2}\right), \tag{4.22}$$

while the Yukawa couplings simplify into $\left(\text{with } \eta^2 = 1\right)$

$$f_{\rm DD}^{s} = e_4 \cos\left(\frac{n}{N}\pi + \frac{ae_4 v}{2}\right), \tag{4.23}$$

$$f_{\rm DD}^{ps} = -e_4 \sin\left(\frac{n}{N}\pi + \frac{ae_4 v}{2}\right). \tag{4.24}$$

5. Anomalous Higgs interaction

In this section we finally discuss how our predictions on the Higgs couplings deviate from the corresponding predictions in the SM. In particular, we now compare the prediction of our model for the Yukawa coupling of the fermion $f_{\rm DD}^{s}$ with the corresponding prediction in the SM, $f_{\rm SM}$. Note that there is no counterpart of $f_{\rm DD}^{ps}$ in the SM, as the Yukawa coupling there is purely of scalar type. We will focus on the sector for the lightest KK mode $n_{\rm min}$ having the smallest $|\sin(\frac{n\pi}{N} + \frac{ae_4 v}{2})|$, since the Yukawa coupling of the lightest fermion is the main interest in the experimental tests.

In the standard model, the Yukawa coupling of a fermion ψ is simply given by the relation

$$f_{\rm SM} = \frac{g}{2}\frac{m_\psi}{M_W}. \tag{5.1}$$

The fermion mass m_ψ corresponds to $m^{(n_{\rm min})}$ in our model, which behaves as a trigonometric function (4.22). Since the violation of the translational invariance is due to the property of the extra space itself, we may naturally expect that in a realistic model the gauge boson mass M_W is also given by a trigonometric function of v like (4.22) for the fermion. In fact, though in the 5D QED the gauge boson does not acquire a mass through the VEV v, in the original paper [10,11], discussing the dimensional deconstruction of non-Abelian gauge theory, the mass eigenvalues for the 4D gauge boson, which acquires a mass due to the VEV through the Hosotani mechanism, has been calculated to be

$$\frac{2}{a}\sin\left(\frac{n\pi}{N}+\frac{agv}{4}\right). \tag{5.2}$$

Since e_4 in our toy model should be identified with $\frac{g}{2}$ in a realistic model, we easily find that the mass eigenvalue (5.2) is identical with that for fermion (4.22). Thus, the fermion actually has the same mass as that of the W boson in a realistic model and the corresponding Yukawa coupling of the fermion in the SM should be $f_{\mathrm{SM}}=\frac{g}{2}$.

Hence the ratio of the predicted scalar-type Yukawa coupling for the lightest fermion in our model to the corresponding Yukawa coupling in the SM becomes (by identifying e_4 with $\frac{g}{2}$):

$$\frac{f^s_{\mathrm{DD}}}{f_{\mathrm{SM}}}=\cos\left(\frac{n_{\min}}{N}\pi+\frac{ae_4v}{2}\right)=\cos\left(\frac{n_{\min}}{N}\pi+x\right), \tag{5.3}$$

where the dimensionless parameter is defined as

$$x\equiv\frac{ae_4v}{2}. \tag{5.4}$$

The ratio in (5.3) deviates from unity and the Yukawa coupling becomes anomalous. It is interesting to note that the predicted Yukawa coupling in the DD scenario is always smaller than the standard model prediction.

Concerning the pseudo-scalar-type Yukawa coupling f^s_{DD}, there is no counterpart in the SM and we just write it by use of x:

$$f^{ps}_{\mathrm{DD}}=-e_4\sin\left(\frac{n_{\min}}{N}\pi+x\right). \tag{5.5}$$

6. Continuum limit and decoupling limit

We finally consider two physically interesting limits

(i) Continuum limit: $N\to\infty$, $a\to 0$ keeping $L=Na=2\pi R$ intact;
(ii) "Decoupling limit": $\frac{M_W}{M_c}\to 0$ keeping N as a finite integer $\left(M_c\equiv\frac{1}{R}\right)$.

Clearly (i) is the limit where the original 5D GHU with S^1 compactification [7] is recovered. Since there is no other source of the violation of the translational invariance except for the latticization, we expect that the anomaly goes away in this continuum limit. Limit (ii) is where the masses of all massive KK modes of the order of M_c (compactification mass scale) are much greater than the weak scale and these massive KK particles are expected to decouple from the low energy effective theory, thus recovering the SM. In our model M_W should be regarded as $\sim e_4v$ and the decoupling limit is equivalent to $e_4vR\sim e_4vL=e_4vNa\sim e_4va\to 0$.

It is now easy to know that in both limits $x=\frac{ae_4v}{2}\to 0$ and therefore $n_{\min}=0$. As is seen in (5.3) and (5.5), in this limit the ratio $\frac{f^s_{\mathrm{DD}}}{f_{\mathrm{SM}}}\to 1$ and $f^{ps}_{\mathrm{DD}}\to 0$. Thus the anomaly just goes away, as we expected.

7. Summary

In this paper we discussed anomalous Higgs interaction with matter fields in the scenario of dimensional deconstruction [10,11], which can accommodate the light Higgs suggested by recent LHC experiments. In the scenario Higgs is regarded as a pseudo-Nambu–Goldstone boson. Though the scenario is formulated in ordinary four-dimensional space-time, the PNGB may be interpreted as the Kaluza–Klein zero mode of the extra space component of the higher-dimensional gauge field. Namely, the scenario of dimensional deconstruction may be interpreted as a "latticized" version of gauge–Higgs unification [3–7].

In our previous paper discussing anomalous Higgs interaction in GHU [20], we pointed out that the interplay between the periodicity of physical observables in the Higgs field and the violation of translational invariance along the extra space plays an essential role in getting the anomalous Higgs interaction. We argued that in the scenario of DD, the interplay between two ingredients is also present. Namely, the periodicity in the Higgs field is guaranteed by the fact that the Higgs field is non-linearly realized in the form of a link variable connecting neighboring lattice sites and the violation of translational invariance is realized by the fact that the extra space is latticized, when DD is regarded as a sort of GHU.

We took the attitude that DD is latticized GHU, and adopted 5D scalar QED and 5D QED as our models.

Among other things, we derived the Yukawa coupling for the lightest KK mode of the matter fermion ("electron") and have confirmed that it is anomalous. Namely, in clear contrast to the SM, the derived Yukawa coupling has two pieces: scalar-type and pseudo-scalar-type couplings. The predicted scalar-type Yukawa coupling in our model deviates from what we expect in the standard model, while the pseudo-scalar-type Yukawa coupling has no counterpart in the SM.

Though the derived anomalous interaction has much similarity to the one in the GHU [14–20], the anomalous interaction in the DD scenario has its own characteristic feature, which is not shared by the GHU scenario. Namely, the anomaly exists even though we do not introduce "Z_2-odd bulk mass" or warp factor. This is because the violation of translational invariance is not due to the presence of the bulk mass term or warp factor, but due to the fact that the extra space itself, on which the theory is constructed, is latticized. Thus the anomalous interactions are expected to appear not only in the sector of matter particles but also in all sectors of the theory. On the other hand, it means that the anomaly goes away (unless we do not introduce another source of the violation of translational invariance such as the bulk mass term) in the continuum limit of lattice spacing $a \to 0$. We have confirmed this property by explicit calculation of the anomalous coupling.

Since our main purpose was to show explicitly that the expected anomalous Higgs interaction in the DD scenario really exists, the adopted models are not enough to describe the real world. To make the theory more realistic we have to achieve several things. Namely, in order to incorporate the SM, the gauge group should be enlarged to non-Abelian symmetry such as SU(3) or SO(5) $\times$ U(1).

In the context of discussing the anomalous Yukawa couplings of quarks and leptons, which should be of immediate interest in the LHC and planned ILC experiments, it is quite important to realize the hierarchical masses of quarks and leptons, though in our simplified model of 5D QED the fermion mass is unique and its Yukawa coupling is of the order of the gauge coupling constant. To realize such hierarchical masses, we have to first solve a problem: how can we realize the orbifold and Z_2-odd bulk mass for the chiral fermion necessary for the hierarchical structure on the latticized extra space? A realistic discussion of anomalous Yukawa coupling will be able to be extended in such a realistic framework.

In spite of these remaining issues, the scenario of DD is attractive since it is a renormalizable 4D gauge theory and realizes at the same time the mechanism of GHU to solve the hierarchy problem [7], by replacing the sum over an infinite number of KK modes in the intermediate state of the quantum correction to the Higgs mass by the sum over just N KK modes (N: the number of lattice sites).

We point out that another interesting scenario of BSM, closely related to DD and GHU, i.e. little Higgs, probably also possesses anomalous Higgs interaction. At least, the non-linearity in the Higgs field should exist, since the Higgs field is regarded to be PNGB and therefore non-linearly realized, just as in the scenario of DD—see (1.7).

Finally, we make a comment on the VEV of the Higgs field. In our analysis, we have assumed that the Higgs field develops a non-vanishing VEV. From the analysis of the effective potential induced at the quantum level, it has been known that in models with bosonic matter like 5D scalar QED the VEV vanishes, at least in the continuum limit $a \to 0$. On the other hand, however, it has been shown in [22] that in 5D QED with a fermionic matter field the Higgs develops a non-vanishing VEV. We thus expect that in a realistic theory with matter fermions, it is quite possible to obtain a desirable VEV of the Higgs field.

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